

Attractor horizons in six-dimensional type IIB supergravity

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Abstract

We consider near horizon geometries of extremal black holes in six-dimensional type IIB supergravity. In particular, we use the entropy function formalism to compute the charges and thermodynamic entropy of these solutions. We also comment on the role of attractor mechanism in understanding the entropy of the Hopf T-dual solutions in type IIA supergravity.

1 Introduction

Warped anti-de Sitter spacetimes in three dimensions (WAdS_3) and their quotients play an important role in the understanding of non-supersymmetric extremal black holes entropy. In particular, WAdS_3 geometries are vacuum solutions of Topologically Massive Gravity, whose stability has been discussed in Ref.[1]. Thus, black hole solutions that are asymptotically WAdS_3 are also expected to exist. Indeed, these black holes are obtained as discrete quotients of WAdS_3 in Ref.[2] in the same way the BTZ black hole [3, 4] is a quotient space of AdS_3 . Furthermore, WAdS_3 appears in the near horizon geometry of spinning extremal black holes and its role is essential in realization of the Kerr/CFT correspondence [5] (see, also, [6] and the references therein).

An important question is whether there is a way to embed WAdS_3 and its quotients in string/ supergravity (SUGRA) theories¹ without a gravitational Chern-Simons term in three dimensions. Once this can be done, one can explore the similarities with the better understood AdS_3 case and try to understand the properties of their dual conformal field theories (CFTs). For example, one can obtain important information about the dual CFT, such as the central charge, by studying the corresponding black holes in the bulk gravity.

Interestingly enough, this embedding can be obtained via the near horizon geometries of extremal black holes [9] (see, also, Ref.[10]) – a discussion of some properties of the dual CFT of these black holes was presented in a nice paper of Anninos [11].

In this letter, we generalize some of the results of Refs.[9, 11] by turning on all moduli. In particular, we explicitly obtain the near horizon geometries of extremal solutions in six-dimensional type IIB SUGRA. We start with a generic near horizon geometry $\text{WAdS}_3 \times S^3$ and show that the physical solutions exist only for a particular fibration of WAdS_3 that, in fact, corresponds to AdS_3 . This result is similar to the one obtained in Refs.[9, 11], where the moduli are turned off. Since these solutions are related by Hopf T-duality [9] to solutions of type IIA SUGRA with an WAdS_3 in the near horizon geometry [12], we expect that their study would shed some light on the properties of their “cousins” in type IIA SUGRA.

When the axions are turned on, the entropy function has flat directions and one may wonder if the arguments of Ref.[11] are still valid in this case. However, it was proven in Ref.[13] that, even if the near horizon background is not uniquely determined by the extremization equations, the entropy is still independent of the asymptotic data. Therefore, due to the attractor mechanism, the results of Ref.[11] can be generalized in this case. That is, since

¹ WAdS_3 can be lifted to a full string theory solution in ten dimensions [7] (see, also, [8] for a related discussion).

the entropies of warped and unwarped solutions match, states with vanishing right-moving temperature in the dual CFT of type IIB solution can still be mapped to thermal states with vanishing right-moving temperature in the dual CFT of type IIA solution.

2 Setup

Instead of looking at the full dimensional reduction of the type IIB action on a six-dimensional torus T^6 [14], we focus on a subset of fields [9] for which one can obtain general enough solutions. The action constructed as a consistent truncation of IIB string theory to six dimensions has the form [9]

$$\begin{aligned} I_{6B} &= \int d^6x \sqrt{-G} \mathcal{L}_{6B} \\ &= \frac{1}{16\pi G_6} \int d^6x \sqrt{-G} \left[R - \frac{1}{2} (\partial\phi_1)^2 - \frac{1}{2} (\partial\phi_2)^2 - \frac{1}{2} e^{2\phi_1} (\partial\chi_1)^2 \right. \\ &\quad \left. - \frac{1}{2} e^{2\phi_2} (\partial\chi_2)^2 - \frac{1}{12} e^{-\phi_1-\phi_2} F^2 - \frac{1}{12} e^{\phi_1-\phi_2} K^2 + \frac{1}{6} \chi_2 F^* H \right], \end{aligned} \quad (2.1)$$

where the six-dimensional manifold is endowed by the metric $G_{\mu\nu}(x)$ and the corresponding scalar curvature is R . The theory contains four scalar fields –two dilatons ϕ_1, ϕ_2 and two axions χ_1, χ_2 ,– the gauge field 2-form A in the NS sector of the theory with the associated field strength 3-form $F = dA$, and also the RR gauge field 2-form B whose field strength 3-form is $K = dB + \chi_1 dA$. We have also introduced a 3-form $H = dB$ and defined a dual 3-form whose components are

$${}^*H^{\mu\nu\lambda} = \frac{1}{3!} \frac{1}{\sqrt{-G}} \epsilon^{\mu\nu\lambda\alpha\beta\gamma} H_{\alpha\beta\gamma}, \quad (2.2)$$

where $\epsilon^{\mu\nu\lambda\alpha\beta\gamma}$ is the constant Levi-Civita tensor density. In our notation the indices are contracted as $FH = F_{\mu\nu\lambda} H^{\mu\nu\lambda}$ and, similarly, $F^*H = F_{\mu\nu\lambda} {}^*H^{\mu\nu\lambda}$.

The equations of motion derived from the action I_{6B} are presented in Appendix A. Since this action is both diffeomorphism and gauge invariant, one can apply the entropy function formalism [13, 15, 16] to obtain the near horizon data of extremal black hole solutions of the theory. The near horizon geometry can be described by a line element that has the form of a product space $\text{WAdS}_3 \times S^3$, and so we consider the following ansatz:

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = v_1 \left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + u (d\varphi + e\rho dt)^2 + v_2 d\Omega_3^2, \quad (2.3)$$

where the local coordinates of the metric of WAdS_3 black hole near the horizon are $t \in \mathbb{R}$, $\rho = r - r_h \in [0, \infty)$ and $\varphi \in [0, 2\pi)$, so that the horizon $r = r_h$ corresponds to $\rho = 0$. The

parameters v_i , $u > 0$, and e are constant. The Ricci tensor ${}^3R^\mu{}_\nu$ and scalar curvature 3R of this three-dimensional submanifold with the coordinates (t, ρ, φ) have the form

$$\text{WAdS}_3 : \quad {}^3R^\mu{}_\nu = \begin{pmatrix} -\frac{2v_1 - ue^2}{2v_1^2} & 0 & 0 \\ 0 & -\frac{2v_1 - ue^2}{2v_1^2} & 0 \\ \frac{e\rho(v_1 - ue^2)}{2v_1^2} & 0 & -\frac{ue^2}{2v_1^2} \end{pmatrix}, \quad {}^3R = \frac{ue^2 - 4v_1}{2v_1^2}, \quad (2.4)$$

where the parameters v_1 , u , and e are arbitrary. The particular case for which $v_1 = ue^2$ corresponds to a spacetime of constant negative curvature with the AdS_3 radius $\ell = 2\sqrt{v_1}$.

For a 3-sphere S^3 of size v_2 , we choose spherical coordinates $\psi, \theta \in [0, \pi)$ and $\alpha \in [0, 2\pi)$, such that

$$d\Omega_3^2 = d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\alpha^2), \quad (2.5)$$

$$\varepsilon(S^3) = \sin^2 \psi \sin \theta d\psi \wedge d\theta \wedge d\alpha, \quad (2.6)$$

$$\text{Vol}(S^3) = \int \varepsilon(S^3) = 2\pi^2. \quad (2.7)$$

The metric $G_{\mu\nu}$ in the local coordinates $x^\mu = (t, \rho, \varphi, \psi, \theta, \alpha)$ then reads

$$G_{\mu\nu} = \text{diag} \left(\begin{bmatrix} -(v_1 - ue^2)\rho^2 & 0 & e\rho \\ 0 & \frac{v_1}{\rho^2} & 0 \\ e\rho & 0 & u \end{bmatrix}, \begin{bmatrix} v_2 & 0 & 0 \\ 0 & v_2 \sin^2 \psi & 0 \\ 0 & 0 & v_2 \sin^2 \psi \sin^2 \theta \end{bmatrix} \right), \quad (2.8)$$

so that the invariant volume element is constructed with $\sqrt{-G} = u^{1/2} v_1 v_2^{3/2} \sin^2 \psi \sin \theta$, and the inverse metric $G^{\mu\nu}$ is

$$G^{\mu\nu} = \text{diag} \left(\begin{bmatrix} -\frac{1}{v_1 \rho^2} & 0 & \frac{e}{v_1 \rho} \\ 0 & \frac{\rho^2}{v_1} & 0 \\ \frac{e}{v_1 \rho} & 0 & \frac{v_1 - ue^2}{uv_1} \end{bmatrix}, \begin{bmatrix} \frac{1}{v_2} & 0 & 0 \\ 0 & \frac{1}{v_2 \sin^2 \psi} & 0 \\ 0 & 0 & \frac{1}{v_2 \sin^2 \psi \sin^2 \theta} \end{bmatrix} \right). \quad (2.9)$$

In order to adopt an ansatz for the gauge fields, let us note that the 3-form $K = dB + \chi_1 dA$ has been defined so that the action explicitly exhibits original symmetries of the eleven-dimensional supergravity [14], but it requires an additional constraint that imposes the Bianchi identity on K . We shall, however, work with the original tensor field strengths $F = dA$ and $H = dB$ that automatically satisfy the Bianchi identities and, in terms of them, $K = H + \chi_1 F$.

An ansatz for the field strengths F and H that possesses the same symmetries as the metric $G_{\mu\nu}$ (that is, $\mathcal{L}_{\xi(i)} F = 0$ and $\mathcal{L}_{\xi(i)} H = 0$), can be written as (for details, see Appendix B)

$$F = e_1 dt \wedge d\rho \wedge d\varphi + p_1 \varepsilon(S^3), \quad (2.10)$$

$$H = e_2 dt \wedge d\rho \wedge d\varphi + p_2 \varepsilon(S^3), \quad (2.11)$$

where e_i and p_i are electric and magnetic charges, respectively.

The ansatz (2.3), (2.10), and (2.11) is similar to the one used in Ref. [17] (see, also, Refs.[18, 19] for similar considerations in AdS).

3 Entropy function

To get the near horizon data, we employ the entropy function formalism presented in Refs. [13, 15, 16] (see, also, Ref. [20]).

We want to calculate the entropy function, \mathcal{E} , defined by

$$\mathcal{E}(\vec{u}, \vec{v}, \vec{e}, \vec{p}) = 2\pi \left(q_i e_i - f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) \right), \quad (3.1)$$

where the electric charges are $\vec{q} = \partial f / \partial \vec{e}$, and

$$f = \int_{\mathcal{H}} d^4x \sqrt{-G} \mathcal{L}_{6B} \quad (3.2)$$

is the action evaluated on the horizon \mathcal{H} on the background (2.3), (2.10) and (2.11) for fixed time. In \mathcal{L}_{6B} , all fields are evaluated at the horizon, as well. In particular, the values of the moduli (scalar fields) at the horizon correspond to a set of parameters $\phi_1(r_h) = u_1$, $\phi_2(r_h) = u_2$, $\chi_1(r_h) = u_3$ and $\chi_2(r_h) = u_4$. Explicitly, we can write

$$f = \frac{1}{16\pi G_6} \int_{\mathcal{H}} d\varphi d\psi d\theta d\alpha \sqrt{-G} \left[R - \frac{1}{12} e^{-u_1-u_2} F^2 - \frac{1}{12} e^{u_1-u_2} (H + \chi_1 F)^2 + \chi_2 F \wedge H \right], \quad (3.3)$$

which, further, taken on a near horizon background, becomes

$$\begin{aligned} f = & \frac{\pi}{8G_6} u^{1/2} v_1 v_2^{3/2} \left[\frac{12}{v_2} - \frac{4}{v_1} + \frac{ue^2}{v_1^2} + e^{-u_1-u_2} \left(\frac{e_1^2}{uv_1^2} - \frac{p_1^2}{v_2^3} \right) \right. \\ & \left. + e^{u_1-u_2} \left(\frac{(e_2 + u_3 e_1)^2}{uv_1^2} - \frac{(p_2 + u_3 p_1)^2}{v_2^3} \right) \right] + \frac{\pi}{4G_6} u_4 (e_1 p_2 - e_2 p_1). \end{aligned} \quad (3.4)$$

The extremization of the entropy function $\mathcal{E}(\vec{u}, \vec{v}, \vec{e}, \vec{p})$ determines the parameters

$$\vec{u} = \{u, u_1, u_2, u_3, u_4\}, \quad \vec{v} = \{v_1, v_2\}, \quad \vec{e} = \{e, e_1, e_2\}, \quad (3.5)$$

in terms of the electric charge \vec{q} and magnetic charge \vec{p} of an extremal black hole,

$$\vec{q} = \{q, q_1, q_2\}, \quad \vec{p} = \{p_1, p_2\}, \quad (3.6)$$

so that, when the attractor mechanism holds, a value of the entropy $S = \mathcal{E}_{\text{ext}}$ does not depend on the asymptotic values of the scalar fields.

The function \mathcal{E} , where f is given by Eq.(3.4), has an extremum for fixed charges \vec{q}, \vec{p} when

$$\frac{\partial f}{\partial \vec{u}} = 0, \quad \frac{\partial f}{\partial \vec{v}} = 0, \quad \frac{\partial f}{\partial \vec{e}} = \vec{q}. \quad (3.7)$$

We solve the attractor equations, which are explicitly given in Appendix A, as follows.

Taking the sum and difference of Eqs.(A.14) and (A.15), we obtain e_1 and e_2 ,

$$\frac{e_1^2}{uv_1^2} = \frac{p_1^2}{v_2^3}, \quad \frac{e_2^2}{uv_1^2} = \frac{p_2^2}{v_2^3}, \quad (3.8)$$

which means that F and H are either self-dual or anti self-dual field strengths, $*F = \mp F$ and $*H = \mp H$, with the dual 3-forms having the components

$$*F = -\frac{p_1 u^{1/2} v_1}{v_2^{3/2}} dt \wedge d\rho \wedge d\varphi - \frac{e_1 v_2^{3/2}}{u^{1/2} v_1} \varepsilon(S^3), \quad (3.9)$$

and similarly for $*H$. Eq.(A.16) determines that F and H must be *simultaneously* dual or anti self-dual. Then, Eq.(A.17) is identically satisfied.

The difference of Eqs.(A.18) and (A.19) leads to a such u that a near horizon geometry in the considered case is always AdS_3 ,

$$v_1 = u e^2. \quad (3.10)$$

The sum of Eqs.(A.19) and (A.20) relates the sizes of two product spaces AdS_2 and S^3 as

$$v_2 = 4v_1, \quad (3.11)$$

as expected in higher dimensions; only in four dimensions and for a vanishing moduli potential the radii of AdS_2 and the sphere are equal.

With the last two results, any of Eqs.(A.18-A.20) gives the size of the AdS_2 space,

$$(8v_1)^2 = e^{-u_1 - u_2} p_1^2 + e^{u_1 - u_2} (p_2 + u_3 p_1)^2. \quad (3.12)$$

Equations (A.21) and (A.22) then determine the dilatons,

$$e^{-2u_2} = \left(\frac{4G_6}{\pi^2} \frac{q_1 - u_3 q_2}{p_2 + u_3 p_1} - u_4 \right) \left(\frac{4G_6}{\pi^2} \frac{q_2}{p_1} + u_4 \right) \quad (3.13)$$

$$e^{-2u_1} = \frac{p_2 + u_3 p_1}{p_1 \left(\frac{4G_6}{\pi^2} q_2 + u_4 p_1 \right)} \left[\frac{4G_6}{\pi^2} (q_1 - u_3 q_2) - u_4 (p_2 + u_3 p_1) \right], \quad (3.14)$$

where $p_1 \neq 0$, $u_3 \neq -\frac{p_2}{p_1}$ and $u_4 \neq -\frac{4G_6}{\pi^2} \frac{q_2}{p_1}$. The values of axions on the horizon remain arbitrary, which means that the entropy function has flat directions. However, as expected, the size of AdS space (3.12) does not depend on them,

$$v_1 = \sqrt{\pm \frac{G_6}{16\pi^2} (p_1 q_1 + p_2 q_2)}. \quad (3.15)$$

Finally, the last unknown parameter is found from Eq.(A.23)

$$e = \sqrt{\pm \frac{p_1 q_1 + p_2 q_2}{8q}}. \quad (3.16)$$

The signs must be chosen so that v_1^2 and e^2 are positive. Thus, depending on the signs of the electric q_i and magnetic p_i charges, only one (dual or anti self-dual) solution is admissible.

To summarize, the entropy function \mathcal{E} has an extremum for the non-trivial values of the parameters \vec{u} , \vec{v} and \vec{e} expressed in terms of \vec{p} and \vec{q} by equations (3.10), (3.11), (3.13 – 3.16) and

$$e_1 = \pm \frac{p_1}{8e}, \quad e_2 = \pm \frac{p_2}{8e}. \quad (3.17)$$

It is important to emphasize that, since $v_1 = ue^2$, the *warped geometry is not allowed* for near horizon solutions in type IIB supergravity when there exists spherical symmetry in the other three angular directions. In the near horizon limit, the 3-forms F and H become (anti) self-dual. Note also that, as expected, the entropy does not depend on the flat directions and it is a function of the physical charges only,

$$\mathcal{E}_{\text{ext}} = 4\pi q e = \sqrt{\pm 2\pi^2 (p_1 q_1 + p_2 q_2) q}. \quad (3.18)$$

When all scalar fields (dilaton and axions) vanish, the moduli equations of the full action (2.1) become (see Appendix A)

$$F^2 = 0, \quad FH = 0, \quad (3.19)$$

$$H^2 = 0, \quad F * H = 0, \quad (3.20)$$

whose only solutions are dual and self-dual gauge field configurations. This is consistent with the results of Ref.[9].

4 Discussion

In this letter, we have explicitly constructed near horizon attractor geometries in type IIB SUGRA theory and generalized some results of Ref.[9] in the presence of the axions. These

configurations are important because they can be related by Hopf T-dualities [9]² to black hole solutions in type IIA SUGRA that have an WAdS₃ in the near horizon geometry, and also could be relevant to understanding the Kerr/CFT correspondence.

Since we have used the entropy function formalism, we were able to exactly obtain the near horizon data and macroscopic entropy. As explained in Section 3, with the axions turned on, the entropy function has flat directions and so the near horizon data do not completely decouple from the asymptotic data of the scalar fields. At first sight, this result could be problematic for the interpretations of the dual CFT of Hopf T-dual type IIA black hole solutions [11]. However, it is known that, due to the long throat of AdS₂, the entropy is still independent of these asymptotic data [13]. A successful application of this method is an indication that the attractor mechanism should also work for the Hopf T-dual solutions. More importantly, the attractor mechanism guarantees that even if the supersymmetry is broken by the Hopf T-duality transformation, one can still compute the statistical entropy for which the microscopic theory is weakly coupled [17, 22].

A further consistent truncation of action (2.1) can be obtained by turning off the axions and a black hole solution in this theory was presented in [11]. The advantage of using the entropy function formalism is that we can explicitly obtain the values of the dilatons at the horizon, which are

$$e^{\phi_1(r_h)} = \sqrt{\frac{p_1 q_2}{q_1 p_2}}, \quad e^{\phi_2(r_h)} = \frac{\pi^2}{4G_6} \sqrt{\frac{p_1 p_2}{q_1 q_2}}.$$

In this case, there are no flat directions and the near horizon data are completely fixed by the electric and magnetic charges due to the attractor mechanism.

Extremal spinning black holes have an WAdS₃ geometry near the horizon and, in principle, one can Kaluza-Klein reduce the theory to obtain AdS₂ and then apply the AdS₂/CFT₁ correspondence. Another proposal to compute the entropy of (non-SUSY) extremal spinning black holes is Kerr/CFT correspondence, though, despite some attempts to embed it in string theory [23], it is not on the same footing with the AdS/CFT duality.

Using the attractor mechanism and the universality of the near horizon geometry, it was shown in [24] that the central charge of the dual CFT of a large class of extremal spinning black holes (computed from the Kerr/CFT correspondence) does not match the central charge obtained from the AdS₂/CFT₁ (though, they are proportional). Hence, even if the final entropy results match, the computations should in fact correspond to two different embeddings in string theory.

Somehow similarly, one can use the observations of Refs.[9, 11] to see that AdS₃ and WAdS₃

²This method was also applied to Taub-Nut spacetimes in Ref.[21].

also correspond to two different embeddings in string theory related by Hopf T-duality. Since the attractor mechanism works in type IIB SUGRA, we expect the same kind of universality of the entropy for non-SUSY extremal black hole solutions (with an WAdS₃ in the near horizon geometry [25]) in type IIA SUGRA [12].

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A Equations of motion for the type IIB action

A more general action of the type (2.1), describing six-dimensional gravity coupled to scalar fields ϕ^i and Abelian gauge field 2-forms A^A with the field-strength 3-forms $F^A = dA^A$, is

$$I[G, \phi, A] = \frac{1}{16\pi G_6} \int d^6x \sqrt{-G} \left[R - \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j - f_{AB}(\phi) F_{\mu\nu\lambda}^A F^{B\mu\nu\lambda} - \tilde{f}_{AB}(\phi) F_{\mu\nu\lambda}^A {}^*F^{B\mu\nu\lambda} \right], \quad (\text{A.1})$$

where the Hodge-dual field strength is ${}^*F^{B\mu\nu\lambda} = \frac{1}{3! \sqrt{-G}} \epsilon^{\mu\nu\lambda\alpha\beta\gamma} F_{\alpha\beta\gamma}^B$ and the metric $G_{\mu\nu}(x)$ lowers and rise the spacetime indices. The equations of motion that extremize the above action are

$$R_{\mu\nu} - \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial_\nu \phi^j = f_{AB} \left(3 F_{\mu\alpha\beta}^A F_\nu^{B\alpha\beta} - \frac{1}{2} G_{\mu\nu} F_{\alpha\beta\gamma}^A F^{B\alpha\beta\gamma} \right), \quad (\text{A.2})$$

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} g_{ij} \partial^\mu \phi^j \right) = \partial_i f_{AB} F_{\mu\nu\lambda}^A F^{B\mu\nu\lambda} + \partial_i \tilde{f}_{AB} F_{\mu\nu\lambda}^A {}^*F^{B\mu\nu\lambda}, \quad (\text{A.3})$$

and

$$\partial_\lambda \left[\sqrt{-G} \left(f_{AB} F^{B\mu\nu\lambda} + \tilde{f}_{AB} {}^*F^{B\mu\nu\lambda} \right) \right] = 0, \quad (\text{A.4})$$

where we denote $\partial_i \equiv \partial/\partial\phi^i$ and the scalar curvature in the Einstein equation has been eliminated using $R = \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j$.

In particular, the field content of the action (2.1) is given by $\phi^i = \{\phi_1, \phi_2, \chi_1, \chi_2\}$ and

$F^A = \{F, H\}$, with the interaction determined by

$$g_{ij} = \text{diag}(1, 1, e^{2\phi_1}, e^{2\phi_2}) , \quad (\text{A.5})$$

$$f_{AB} = \frac{e^{\phi_1 - \phi_2}}{12} \begin{pmatrix} 1 + e^{-2\phi_1} & \chi_1 \\ \chi_1 & 1 \end{pmatrix} , \quad \tilde{f}_{AB} = \frac{\chi_2}{12} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} . \quad (\text{A.6})$$

The equations of motion in this case are as follows. The Einstein equations read

$$\begin{aligned} R_{\mu\nu} = & \frac{1}{2} (\partial_\mu \phi_1 \partial_\nu \phi_1 + \partial_\mu \phi_2 \partial_\nu \phi_2 + e^{2\phi_1} \partial_\mu \chi_1 \partial_\nu \chi_1 + e^{2\phi_2} \partial_\mu \chi_2 \partial_\nu \chi_2) \\ & + \frac{1}{12} e^{-\phi_1 - \phi_2} \left(3F_{\mu\alpha\beta} F_\nu{}^{\alpha\beta} - \frac{1}{2} G_{\mu\nu} F^2 \right) \\ & + \frac{1}{12} e^{\phi_1 - \phi_2} \left(3K_{\mu\alpha\beta} K_\nu{}^{\alpha\beta} - \frac{1}{2} G_{\mu\nu} K^2 \right) . \end{aligned} \quad (\text{A.7})$$

The field equations for dilations are

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} G^{\mu\nu} \partial_\nu \phi_1 \right) - e^{2\phi_1} (\partial \chi_1)^2 = -\frac{1}{12} (e^{-\phi_1 - \phi_2} F^2 - e^{\phi_1 - \phi_2} K^2) , \quad (\text{A.8})$$

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} G^{\mu\nu} \partial_\nu \phi_2 \right) - e^{2\phi_2} (\partial \chi_2)^2 = -\frac{1}{12} (e^{-\phi_1 - \phi_2} F^2 + e^{-\phi_1 - \phi_2} K^2) , \quad (\text{A.9})$$

whereas for the axions they have the form

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} G^{\mu\nu} e^{2\phi_1} \partial_\nu \chi_1 \right) = \frac{1}{6} e^{\phi_1 - \phi_2} F K , \quad (\text{A.10})$$

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} G^{\mu\nu} e^{2\phi_2} \partial_\nu \chi_2 \right) = \frac{1}{6} {}^* F H . \quad (\text{A.11})$$

Finally, the equations of motion for the gauge fields $A_{\mu\nu}$ and $B_{\mu\nu}$ can be written as

$$\frac{1}{\sqrt{-G}} \partial_\mu \left[\sqrt{-G} (e^{-\phi_1 - \phi_2} F^{\mu\nu\lambda} + \chi_1 e^{\phi_1 - \phi_2} K^{\mu\nu\lambda}) \right] = \partial_\mu \chi_2 {}^* H^{\mu\nu\lambda} , \quad (\text{A.12})$$

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} e^{\phi_1 - \phi_2} K^{\mu\nu\lambda} \right) = -\partial_\mu \chi_2 {}^* F^{\mu\nu\lambda} . \quad (\text{A.13})$$

In the near horizon limit when the background becomes (2.3), the attractor equations are obtained by the extremization of the entropy function given by Eqs.(3.1) and (3.4). Varying it in the moduli gives

$$\delta u_1 : \quad 0 = -e^{-u_1 - u_2} \left(\frac{e_1^2}{uv_1^2} - \frac{p_1^2}{v_2^3} \right) + e^{u_1 - u_2} \left(\frac{e_2^2 + u_3^2 e_1^2}{uv_1^2} - \frac{p_2^2 + u_3^2 p_1^2}{v_2^3} \right) , \quad (\text{A.14})$$

$$\delta u_2 : \quad 0 = e^{-u_1 - u_2} \left(\frac{e_1^2}{uv_1^2} - \frac{p_1^2}{v_2^3} \right) + e^{u_1 - u_2} \left(\frac{e_2^2 + u_3^2 e_1^2}{uv_1^2} - \frac{p_2^2 + u_3^2 p_1^2}{v_2^3} \right) , \quad (\text{A.15})$$

$$\delta u_3 : \quad 0 = u_3 \left(\frac{e_1^2}{uv_1^2} - \frac{p_1^2}{v_2^3} \right) + \left(\frac{e_1 e_2}{uv_1^2} - \frac{p_1 p_2}{v_2^3} \right) , \quad (\text{A.16})$$

$$\delta u_4 : \quad 0 = e_1 p_2 - e_2 p_1 , \quad (\text{A.17})$$

and varying it in the metric parameters δu , δv_1 and δv_2 gives, respectively,

$$\frac{12}{v_2} - \frac{4}{v_1} + \frac{3ue^2}{v_1^2} = e^{-u_1-u_2} \left(\frac{e_1^2}{uv_1^2} + \frac{p_1^2}{v_2^3} \right) + e^{u_1-u_2} \left[\frac{(e_2 + u_3 e_1)^2}{uv_1^2} + \frac{(p_2 + u_3 p_1)^2}{v_2^3} \right], \quad (\text{A.18})$$

$$\frac{12}{v_2} - \frac{ue^2}{v_1^2} = e^{-u_1-u_2} \left(\frac{e_1^2}{uv_1^2} + \frac{p_1^2}{v_2^3} \right) + e^{u_1-u_2} \left[\frac{(e_2 + u_3 e_1)^2}{uv_1^2} + \frac{(p_2 + u_3 p_1)^2}{v_2^3} \right], \quad (\text{A.19})$$

$$\frac{4}{v_2} - \frac{4}{v_1} + \frac{ue^2}{v_1^2} = -e^{-u_1-u_2} \left(\frac{e_1^2}{uv_1^2} + \frac{p_1^2}{v_2^3} \right) - e^{u_1-u_2} \left[\frac{(e_2 + u_3 e_1)^2}{uv_1^2} + \frac{(p_2 + u_3 p_1)^2}{v_2^3} \right]. \quad (\text{A.20})$$

Finally, the extremization in \vec{e} leads to

$$\delta e_1 : \quad q_1 = \frac{\pi}{4G_6} \left[e^{-u_1-u_2} \frac{e_1 v_2^{3/2}}{v_1 u^{1/2}} + e^{u_1-u_2} \frac{(e_2 + u_3 e_1) u_3 v_2^{3/2}}{v_1 u^{1/2}} + u_4 p_2 \right], \quad (\text{A.21})$$

$$\delta e_2 : \quad q_2 = \frac{\pi^2}{4G_6} \left[e^{u_1-u_2} \frac{v_2^{3/2} (e_2 + u_3 e_1)}{v_1 u^{1/2}} - u_4 p_1 \right], \quad (\text{A.22})$$

$$\delta e : \quad q = \frac{\pi^2}{4G_6} \frac{eu^{3/2}v_2^{3/2}}{v_1}. \quad (\text{A.23})$$

B Near horizon isometries and invariant p -forms

We want to find the ansätze of p -forms

$$F_{(p)} = \frac{1}{p!} F_{\mu_1 \dots \mu_p} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}, \quad p = 2, 3, 4, \quad (\text{B.1})$$

that possess the same symmetries as a 6-dimensional spacetime that is a direct product of WAdS_3 and S^3 , as given by the metric $G_{\mu\nu}$, Eq.(2.3). Since spacetime is a product space, the p -form with $p \leq 3$ is a sum of two p -forms, each one defined on a respective three-dimensional subspace,

$$F_{(p)} = F_{(p)}|_{\text{WAdS}_3} + F_{(p)}|_{S^3}. \quad (\text{B.2})$$

For this purpose, we find the isometries of $G_{\mu\nu}$. In the coordinates $x^\mu = (t, \rho, \varphi, \psi, \theta, \alpha)$ and for $v_1 - ue^2 \neq 0$, the Killing equation

$$\mathcal{L}_\xi G_{\mu\nu} = \partial_\mu \xi^\lambda G_{\lambda\nu} + \partial_\nu \xi^\lambda G_{\mu\lambda} + \xi^\lambda \partial_\lambda G_{\mu\nu} = 0, \quad (\text{B.3})$$

has 10 linearly independent solutions $\xi^{(i)} = \xi^{(i)\mu} \partial_\mu$, that correspond to near horizon isometries

$$\begin{aligned}
\xi^{(1)} &= \partial_t, \\
\xi^{(2)} &= t\partial_t - \rho\partial_\rho, \\
\xi^{(3)} &= -\left(t^2 + \frac{1}{\rho^2}\right)\partial_t + 2t\rho\partial_\rho + \frac{2e}{\rho}\partial_\varphi, \\
\xi^{(4)} &= \partial_\varphi, \\
\xi^{(5)} &= -\cos\alpha\partial_\theta + \cot\theta\sin\alpha\partial_\alpha, \\
\xi^{(6)} &= \sin\alpha\partial_\theta + \cot\theta\cos\alpha\partial_\alpha, \\
\xi^{(7)} &= \partial_\alpha, \\
\xi^{(8)} &= -\cos\theta\partial_\psi + \cot\psi\sin\theta\partial_\theta, \\
\xi^{(9)} &= -\cos\alpha\sin\theta\partial_\psi + \cot\psi\left(-\cos\alpha\cos\theta\partial_\theta + \frac{\sin\alpha}{\sin\theta}\partial_\alpha\right), \\
\xi^{(10)} &= \sin\alpha\sin\theta\partial_\psi + \cot\psi\left(\sin\alpha\cos\theta\partial_\theta + \frac{\cos\alpha}{\sin\theta}\partial_\alpha\right).
\end{aligned}$$

The isometry algebra $SL(2, \mathbb{R}) \times U(1) \times SO(4)$ is generated by the operators

$$\begin{aligned}
SL(2, \mathbb{R}) : \quad & \mathbf{L}_1 = \frac{1}{2}(\xi^{(1)} - \xi^{(3)}), \quad & SO(4) : \quad & \mathbf{J}_{12} = \xi^{(7)}, \quad & \mathbf{J}_{23} = -\xi^{(5)}, \\
& \mathbf{L}_2 = \frac{1}{2}(\xi^{(1)} + \xi^{(3)}), \quad & & \mathbf{J}_{13} = -\xi^{(6)}, \quad & \mathbf{J}_{24} = -\xi^{(9)}, \\
& \mathbf{L}_3 = \xi^{(2)}, \quad & & \mathbf{J}_{14} = -\xi^{(10)}, \quad & \mathbf{J}_{34} = \xi^{(8)}, \\
U(1) : \quad & \mathbf{G} = \xi^{(4)},
\end{aligned}$$

that satisfy the Lie brackets $[\mathbf{L}_k, \mathbf{L}_l] = \epsilon_{klm} \mathbf{L}^m$, and $[\mathbf{J}_{ab}, \mathbf{J}_{cd}] = \delta_{ad} \mathbf{J}_{bc} - \delta_{bd} \mathbf{J}_{ac} - \delta_{ac} \mathbf{J}_{bd} + \delta_{bc} \mathbf{J}_{ad}$.

Having a complete set of the Killing vectors $\{\xi^{(i)} | i = 1, \dots, 10\}$, for an each p -form $F_{(p)}$, we have to solve the set of equations

$$\mathcal{L}_{\xi^{(i)}} F_{\mu_1 \dots \mu_p} \equiv \partial_{\mu_1} \xi^{(i)\nu} F_{\nu \mu_2 \dots \mu_p} + \partial_{\mu_2} \xi^{(i)\nu} F_{\mu_1 \nu \mu_3 \dots \mu_p} + \dots + \partial_{\mu_p} \xi^{(i)\nu} F_{\mu_1 \dots \mu_{p-1} \nu} + \xi^{(i)\nu} \partial_\nu F_{\mu_1 \dots \mu_p} = 0.$$

It is straightforward to show that components of a p -form invariant under the Lie-dragging along the vectors ∂_t , ∂_φ and ∂_α , do not depend on the coordinates t , φ and α .

In particular, we are interested in a 3-form

$$F_{(3)} = F_{t\rho\varphi}(\rho) dt \wedge d\rho \wedge d\varphi + F_{\psi\theta\alpha}(\psi, \theta) d\psi \wedge d\theta \wedge d\alpha. \quad (\text{B.4})$$

Invariance under $\xi^{(2)}$ gives

$$\mathcal{L}_{\xi^{(2)}} F_{t\rho\varphi} = -\rho\partial_\rho F_{t\rho\varphi} = 0 \quad \Rightarrow \quad F_{t\rho\varphi} = e_1 = \text{Const}, \quad (\text{B.5})$$

and invariance under the action of $\xi^{(5)}$ leads to

$$\mathcal{L}_{\xi^{(5)}} F_{\psi\theta\alpha} = \cos\alpha (\cot\theta F_{\psi\theta\alpha} - \partial_\theta F_{\psi\theta\alpha}) = 0 \quad \Rightarrow \quad F_{\psi\theta\alpha}(\psi, \theta) = A(\psi) \sin\theta. \quad (\text{B.6})$$

Finally, the $\xi^{(8)}$ -invariance gives

$$\mathcal{L}_{\xi^{(8)}} F_{\psi\theta\alpha} = \sin\theta \cos\theta (2A \cot\psi - \partial_\psi A) = 0 \quad \Rightarrow \quad A(\psi) = p_1 \sin^2\psi, \quad (\text{B.7})$$

where $p_1 = \text{Const.}$ All other Killing vectors leave this solution for $F_{\psi\theta\alpha}$ invariant.

Therefore, the most general 3-form possessing the same isometries as the metric $G_{\mu\nu}$ is

$$F_{(3)} = e_1 dt \wedge d\rho \wedge d\varphi + p_1 \varepsilon(S^3), \quad (\text{B.8})$$

where $\varepsilon(S^3) = \sin^2\psi \sin\theta d\psi \wedge d\theta \wedge d\alpha$.

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